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TOPICAL ISSUE

Structural Spectral Methods of Solving Continuous Generalized Lyapunov Equation

I. B. Yadykin[*,a] **and I. A. Galyaev**[*,b]

[*] *Trapeznikov Institute of Control Sciences, Russian Academy of Sciences, Moscow, Russia*
e-mail: [a] *Jad@ipu.ru,* [b] *ivan.galyaev@yandex.ru*

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Abstract—Methods and algorithms for obtaining analytical formulas for spectral decompositions of gramians for bilinear multi-connected continuous stationary stable systems with a simple spectrum are developed. A guaranteed limited area of distribution of methods for solving and analyzing linear control systems to a class of bilinear systems is found. New sufficient conditions for BIBO stability of bilinear systems are developed. The obtained spectral decompositions of solutions by the spectrum of the dynamics matrix of the linear part, as well as the spectrum and residues of the images of actions, allow us to estimate their influence on the stability and dynamic characteristics of the bilinear system.

Keywords: spectral decompositions of gramians, Laplace transform, inverse Laplace transform, generalized Lyapunov equation, H_2-norm, controllability gramian

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1. INTRODUCTION

Maintaining uninterrupted, stable operation of the power system is one of the most important tasks of the electric power industry. Loss of stability of the power system leads to voltage failure and power outages for energy consumers. One of the approaches to describing the functioning processes of a real power system is the creation of a simplified physical model consisting of a large number of oscillatory systems, which are elastically connected groups of generators. As a rule, oscillatory subsystems have different resonant frequencies. In the event of resonance of certain subsystems or disconnection of generators, unstable oscillatory subsystems begin to interact, which leads to the development of unstable processes in the entire power system.

One of the effective methods for analyzing the static stability of power systems is the gramian method. Analysis of the gramian controllability of a linear model of a power system provides information on the distribution of power in the electric network, on the influence of individual groups of generators and consumers on the throughput of a particular section of the network [1]. The assessment of the ultimate stability boundaries is based on the assessment of the energy accumulated in the group of weakly stable modes. From physical considerations, it becomes clear that the growth of this energy means that the power system is approaching the stability boundary. If the transfer function of its linear model is known, the oscillation energy can be estimated by the square of the H_2-norm of the transfer function, which can be calculated by solving the Lyapunov equations and calculating the energy functionals [2–4]. A blackout is an example of a severe systemic accident in a power system, the degree of threat of which can be calculated using the gramian method. However, it is based on the use of a linearized model and does not allow analyzing stability during short circuits on lines, which requires taking into account the factors of nonlinearities of the model.

The choice of a bilinear model of the power system allows one to take into account the nonlinearities of interactions. For such a model, the calculation of the H_2-norm of the operator is based on the expansion of the resolvent of the linear system dynamics matrix into simple fractions in the complex domain. There are also iterative algorithms for calculating the square of the H_2-norm of controllability gramians. Bilinear models of power systems are used to analyze the static stability of power systems [1, 5]. To solve the problem of stability analysis [6, 7], the multidimensional Laplace transform is used. The first attempts at an alternative solution using the gramians method for nonlinear models of dynamic systems were associated with the scientific direction of dimensionality reduction, as well as the calculation of kinetic and accumulated energy. In [8], an iterative method for calculating the square of the H_2-norm for the bilinear system operator was first developed. In [9–12], Volterra functional series and multidimensional transfer functions are used to synthesize nonlinear control systems. The gramian method is used to calculate spectral decompositions of controllability, observability and cross-gramians from solution matrices of the Lyapunov and Sylvester equations for continuous and discrete systems with simple and multiple spectra. The gramian method for calculating virtual energy balances based on the spectral decomposition of the square of the H_2-norm for the transfer function of the system is proposed in [13]. Energy indicators for energy balance anomalies are determined and their expressions are obtained in terms of quadratic complex-valued forms. Comparison of the absolute values of these forms allows us to identify balance anomalies and, what is equally important, to point out specific devices causing the anomalies. For the tasks of monitoring the stability of electric power systems, these anomalies determine the severity of the threat of instability and the direction of development of a possible cascade accident. For the tasks of technical diagnostics, they determine possible degradation failures of technical devices [14].

The main contribution of the work can be defined as a new method of spectral decomposition of Volterra matrix series for the purpose of calculating the gramian functionals and the energy of a bilinear system, calculating the H_2-norm for a bilinear system based on the decomposition of the resolvent of the linear system dynamics matrix into simple fractions in the complex domain. In addition, iterative algorithms for calculating the square of the H_2-norm of controllability gramians for a continuous bilinear system based on the use of the direct and inverse Laplace transform at each iteration step are developed.

2. PROBLEM STATEMENT

A stable continuous stationary bilinear dynamic MIMO system is considered [19]

$$\Sigma_2\colon \begin{cases} \dfrac{dx}{dt} = Ax\left(t\right) + \displaystyle\sum_{\gamma=1}^{m} N_\gamma x\left(t\right) u_\gamma\left(t\right) + Bu\left(t\right), \\ y\left(t\right) = Cx\left(t\right), \end{cases} \tag{2.1}$$

where $x(t) \in R^n$, $u(t) \in R^m$, $y(t) \in R^m$, $u_\gamma(t)$ is γth component $u(t)$.

For the system (2.1) the linear part is defined

$$\Sigma_1\colon \begin{cases} \dfrac{dx}{dt} = Ax\left(t\right) + Bu\left(t\right), \\ y\left(t\right) = Cx\left(t\right). \end{cases} \tag{2.2}$$

A valid representation of the gramian controllability of a bilinear system by means of a matrix Volterra series is given by [1]

$$
\begin{aligned}
&P_1(t_1) = e^{At_1}B,\\
&P_i(t_1,\ldots,t_i) = e^{At_i}\left[N_1P_{i-1}N_2P_{i-1}\ldots N_mP_{i-1}\right], \quad i = 2,3,\ldots,\\
&P = \sum_{i=1}^{\infty}\int_0^{\infty}\ldots\int_0^{\infty} P_i(t_1,\ldots,t_i)P_i^{\mathrm{T}}(t_1,\ldots,t_i)dt_1\ldots dt_i.
\end{aligned} \tag{2.3}
$$

For the system (2.1), two representations of the generalized Lyapunov equation (GLE) are known through the controllability and observability grammians, respectively

$$AP + PA^T + \sum_{\gamma=1}^{m} N_\gamma P N_\gamma^{\mathrm{T}} = -BB^{\mathrm{T}}, \tag{2.4}$$

$$A^{\mathrm{T}}Q + QA + \sum_{\gamma=1}^{m} N_\gamma Q N_\gamma^{\mathrm{T}} = -C^{\mathrm{T}}C. \tag{2.5}$$

Lemma 1 [1]. *If a sequence of vectors* $\{x_i(t)\}$ *of solutions to the differential equations of the system* (2.1), *in which the control vector is defined on the space of continuous real vectors* $U^m\,(I)$ *on a finite interval* $I = (0,T)$ *with the same initial conditions as for the linear system* (2.2) *is true*

$$\dot{x}_0 = Ax_0 + Bu, \tag{2.6}$$

$$\dot{x}_i = Ax_i + \sum_{\gamma=1}^{m} N_\gamma x_{i-1}u_\gamma + Bu, \quad i = 1,2,\ldots, \tag{2.7}$$

$$x_i\,(0) = x\,(0)\,, \quad i = 1,2,\ldots.$$

Then for each vector $u\,(t) \in U^p\,(I)$ *the sequence of vectors* $\{x_i(t)\}$ *of solutions of the systems* (2.6)–(2.7) *converges uniformly on* I *to the solution of the bilinear system* (2.1) – $\{x(t)\}$.

Denote the residual vector $z_i\,(t) = x\,(t) - x_i\,(t)$, $i = 1,2,\ldots$.

Then the equalities are valid

$$z_i\,(t) = \int_0^t e^{A(t-\tau)}\sum_{\gamma=1}^{m} N_\gamma z_{i-1}(\tau)\,u_\gamma\,(\tau)\,d\tau, \quad i = 1,2,\ldots. \tag{2.8}$$

For zero initial conditions

$$x_i\,(0) = x\,(0) = 0, \quad z_i\,(0) = 0, \quad i = 1,2,\ldots,$$

the solution to the system of differential equations (2.6) will take the form

$$x_0\,(t) = \int_0^t e^{A(t-\tau)}Bu\,(\tau)\,d\tau. \tag{2.9}$$

Theorem 1 [24]. *The Volterra series* (2.3) *converges on the time interval* $[0,\inf)$ *for any bounded input signal if the following two conditions are satisfied:*

1. *Matrix* A *is stable, i.e.* $\Lambda(A) \subset C^-$.

2. *The matrices* N_γ *are quite limited, i.e.* $\sum_{\gamma=1}^{m}||N_\gamma|| < \frac{\mu}{cM}$, *where two constants* $\mu > 0$ *and* $c > 0$ *are such that*

$$||e^{At}||_2 \leqslant ce^{-\mu t/2}, \quad t \geqslant 0.$$

In this paper, a new comprehensive approach to constructing solutions $z_i\,(t)$ is proposed with the aim of developing new iterative solution algorithms and constructive verifiable criteria for convergence of solutions on the half-interval $[0,\infty)$. A new methodology for constructing a solution is proposed to solve the problem:

1. At the first iteration, perform EVD decomposition of the linear part dynamics matrix.

2. Calculate the solution for the vector elements at each step in the time and frequency domain using the direct and inverse Laplace transform based on spectral decomposition and aggregation of the vector elements.

3. Form functional sequences of elements of the state vector of a bilinear system and construct integral inequalities to construct their majorants.

4. Obtain criteria for convergence of elements of solutions on the half-interval $[0,\infty)$ and, based on them, perform a BIBO stability analysis of the bilinear system.

3. MAIN RESULTS

In this formulation, we assume that the matrix A is stable and has a simple spectrum, $m = 1$, and the function $u\,(t)$ is bounded on the half-interval $[0,\infty)$

$$\int\limits_0^\infty |u(\tau)|\,d\tau \leqslant M > 0. \tag{3.1}$$

If all eigenvalues s_r of matrix A are distinct, then there exists a non-degenerate coordinate transformation

$$\begin{gathered} x = Tx_d, \quad z = Tz_d, \quad \dot{zx}_d = A_d zx_d + B_d u, \quad y_d = C_d x_d, \\ A_d = T^{-1}AT, \quad B_d = T^{-1}B, \quad C_d = C\mathrm{T}, \quad Q_d = T^{-1}BB^{\mathrm{T}}T^{-\mathrm{T}}, \end{gathered} \tag{3.2}$$

or

$$A_d = \begin{bmatrix} u_1 & u_2 & \dots & u_n \end{bmatrix} \begin{bmatrix} s_1 & 0 & 0 & 0 \\ 0 & s_2 & 0 & 0 \\ \dots & \dots & \dots & \dots \\ 0 & 0 & \dots & s_n \end{bmatrix} \begin{bmatrix} \nu_1^* \\ \nu_2^* \\ \vdots \\ \nu_n^* \end{bmatrix}; \quad TV = VT = I.$$

Where the matrix T is composed of the left eigenvectors u_i, and the matrix $T^{-1} = V$ is composed of the right eigenvectors ν_i^*, corresponding to the eigenvalue s_i.

Consider the process of sequential construction of solutions $z_d(t)$.

Step one. In the time domain, the solution is given by the equation (2.9). For the element "φ" of the diagonalized system, this equation is

$$z_{d\varphi}^{(1)}\,(t) = \int\limits_0^t e^{s_\varphi(t-\tau)} b_\varphi u\,(\tau)\,d\tau, \quad t \in [0,\infty),$$

from where, taking into account the condition $s_\varphi \in \mathrm{C}^-$, the inequality follows

$$\left|z_{d\varphi}^{(1)}\,(t)\right| \leqslant \max_\varphi |b_\varphi|\,M, \quad \varphi = 1,2,\dots,n, \quad t \in [0,\infty).$$

Since (3.1) implies the existence of the image $u(s)$, in the frequency domain the exact solution has the form

$$z_{d\varphi}^{(1)}\,(s) = (s - s_\varphi)^{-1} b_\varphi u\,(s)\,.$$

Step two. According to (2.8), the solution in the time domain is

$$z_{d\varphi}^{(2)}(t) = \int_0^t e^{s_\varphi (t-\tau)} N z_{d\varphi}^{(1)}(\tau)\, u(\tau)\, d\tau, \quad \varphi = 1, 2, \ldots, n, \quad t \in [0, \infty). \tag{3.3}$$

The image of the convolution integral (3.3) has the form

$$z_{d\varphi}^{(2)}(s) = (s - s_\varphi)^{-1} \mathcal{L}\left[N z_{d\varphi}^{(1)}(\tau)\, u(\tau)\right].$$

Since all eigenvalues s_φ are in the left half-plane, the following inequalities hold

$$\left|e^{s_\varphi(t-\tau)}\right| < 1, \quad \tau \in [0,\infty),$$

$$\left|z_{d\varphi}^{(2)}(t)\right| \leqslant \int_0^t \left|N b_\varphi u^2(\tau)\right| d\tau, \quad \varphi = 1, 2, \ldots, n, \quad t \in [0, \infty). \tag{3.4}$$

If the condition (3.1) is satisfied, the function $u(t)$ is Laplace transformable. Assume that its image $u(s)$ is a rational algebraic fraction with l simple poles

$$u(s) = \frac{A(s)}{B(s)} = \frac{A(s)}{\prod_{k=1}^{l} (s - s_k)}.$$

The expansion of this function into simple fractions with complex coefficients has the form

$$u(s) = \sum_{k=1}^{l} R_k^u (s - s_k)^{-1}, \quad R_k^u = \frac{A(s_k)}{\dot{B}(s_k)},$$

where R_k^u is the residue of the function $u(s)$ at its pole. Based on the theorem on the multiplication of two functions in the time domain, we have

$$\mathcal{L}\left[e_i^{\mathrm{T}} N z_{d\varphi}^{(1)}(\tau)\, u(\tau)\right] = \sum_{\varphi=1}^{n} \sum_{k=1}^{l} n_{i\varphi} b_\varphi R_k^u u(s - s_k), \tag{3.5}$$

$$\mathcal{L}^{-1}\left\{\mathcal{L}\left[e_i^{\mathrm{T}} N z_{d\varphi}^{(1)}(\tau)\, u(\tau)\right]\right\} = \sum_{\varphi=1}^{n} \sum_{k=1}^{l} n_{i\varphi} b_\varphi R_k^u u(t) e^{s_k t}. \tag{3.6}$$

From (3.6), taking into account the stability of the linear part of the bilinear system, the inequality follows

$$\left|\mathcal{L}^{-1}\left\{\mathcal{L}\left[e_i^{\mathrm{T}} N z_{d\varphi}^{(1)}(\tau)\, u(\tau)\right]\right\}\right| \leqslant \left|\sum_{\varphi=1}^{n} \sum_{k=1}^{l} n_{i\varphi} b_\varphi R_k^u\right| |u(t)|, \ \forall \varphi, i; \quad \forall t \in [0,\infty).$$

Taking into account the last inequality, the inequality (3.4) takes the form

$$\left|z_{di}^{(2)}(t)\right| \leqslant \left|\sum_{\varphi=1}^{n} \sum_{k=1}^{l} n_{i\varphi} b_\varphi R_k^u\right| \int_0^t |u(\tau)| d\tau, \quad \forall \varphi, i = 1, 2, \ldots, n, \quad t \in [0, \infty). \tag{3.7}$$

Step three. According to the general formula, the solution in the time domain is

$$z_{d\varphi}^{(3)}(t) = \int_0^t e^{s_\varphi (t-\tau)} N z_{d\varphi}^{(2)}(\tau)\, u(\tau)\, d\tau, \quad \forall \varphi, \varphi = 1, 2, \ldots, n, \quad t \in [0, \infty). \tag{3.8}$$

The image of the convolution integral (3.8) takes the form

$$z_{d\varphi}^{(3)}(s) = (s - s_\varphi)^{-1} \mathcal{L}\left[N z_{d\varphi}^{(2)}(\tau)\, u(\tau)\right].$$

Since all eigenvalues s_φ are in the left half-plane, the following inequalities hold

$$\left| e^{s_\varphi (t-\tau)} \right| < 1, \quad \forall t, \tau \in [0,\infty),$$

$$\left| z_{d\varphi}^{(3\ )}(t) \right| \leqslant \int\limits_0^t \left| N b_\varphi u^2(\tau) \right| d\tau, \quad \forall \varphi, \varphi = 1, 2, \ldots, n, \quad t \in [0, \infty). \tag{3.9}$$

Based on the theorem on the multiplication of two functions in the time domain, the equations are valid

$$\mathcal{L}\left[e_i^{\mathrm{T}} N z_{d\varphi}^{(2)}(\tau)\, u(\tau)\right] = \sum_{\varphi=1}^{n} \sum_{k=1}^{l} n_{i\varphi} b_\varphi R_k^u u(s - s_k), \tag{3.10}$$

$$\mathcal{L}^{-1}\left\{\mathcal{L}\left[e_i^{\mathrm{T}} N z_{d\varphi}^{(2)}(\tau)\, u(\tau)\right]\right\} = \sum_{\varphi=1}^{n} \sum_{k=1}^{l} n_{i\varphi} b_\varphi R_k^u u(t) e^{s_k t}. \tag{3.11}$$

From (3.11), taking into account the stability of the linear part of the bilinear system, the inequality follows

$$\left| \mathcal{L}^{-1}\left\{\mathcal{L}\left[e_i^{\mathrm{T}} N z_{d\varphi}^{(2)}(\tau)\, u(\tau)\right]\right\} \right| \leqslant \left| \sum_{\varphi=1}^{n} \sum_{k=1}^{l} n_{i\varphi} b_\varphi R_k^u \right| |u(t)|, \ \forall \varphi, i; \quad \forall t \in [0,\infty).$$

The equation (3.9) is transformed

$$\begin{gathered} \left| z_{d\rho}^{(3)}(t) \right| \leqslant \left| \sum_{\psi=1}^{n} \sum_{k=1}^{l} n_{\psi\varphi} R_k^u \right| \left| \sum_{\varphi=1}^{n} \sum_{k=1}^{l} n_{\psi\varphi} b_\varphi R_k^u \right| M, \\ \forall \varphi, i = 1, 2, \ldots, n, \quad t \in [0, \infty), \end{gathered} \tag{3.12}$$

or

$$\left| z_{d\rho}^{(3)}(t) \right| \leqslant nl \max_{\psi,\varphi} |n_{\psi\varphi}| \max_k |R_k^u| \left| z_{d\rho}^{(2)}(t) \right|, \ t \in [0, \infty).$$

The following recurrent inequalities are valid at all subsequent steps $j = 4, 5, \ldots$

$$\left| z_{d\rho}^{(j)}(t) \right| \leqslant nl \max_{\psi,\varphi} |n_{\psi\varphi}| \max_k |R_k^u| \left| z_{d\rho}^{(j-1)}(t) \right|, \ t \in [0, \infty). \tag{3.13}$$

Via the method of mathematical induction, the inequality is valid for $j = 3$.

Assume that it is valid for the step $j - 1$

$$\left| z_{d\rho}^{(j-1)}(t) \right| \leqslant nl \max_{\psi,\varphi} |n_{\psi\varphi}| \max_k |R_k^u| \left| z_{d\rho}^{(j-2)}(t) \right|, \quad t \in [0, \infty),$$

$$\left| z_{d\rho}^{(j-2)}(t) \right| \leqslant \left\{ nl \max_{\psi,\varphi} |n_{\psi\varphi}| \max_k |R_k^u| \right\}^{j-3} \left| \sum_{\varphi=1}^{n} \sum_{k=1}^{l} n_{i\varphi} b_\varphi R_k^u \right| M. \tag{3.14}$$

In accordance with the general algorithm (2.8)

$$z_{d\rho}^{(j)}(t) = \int\limits_0^t e^{s_\rho (t-\tau)} N z_{d\rho}^{(j-1)}(\tau)\, u(\tau)\, d\tau, \quad \forall \varphi, \varphi = 1, 2, \ldots, n, \quad t \in [0, \infty). \tag{3.15}$$

Due to the assumption of stability of the linear part, the inequality is true

$$\left|z_{d\rho}^{(j)}(t)\right| \leqslant \left|\int_0^t N z_d^{(j-1)}(\tau)\, u(\tau)\, d\tau\right|, \quad \forall\varphi, \varphi = 1, 2, \ldots, n, \quad t \in [0, \infty). \tag{3.16}$$

On the other hand, the assessment is fair

$$\left|\mathcal{L}^{-1}\left\{\mathcal{L}\left[e_\rho^{\mathrm{T}} N z_d^{(j-1)}(\tau)\, u(\tau)\right]\right\}\right| \leqslant \left|\sum_{\varphi=1}^{n}\sum_{k=1}^{l} n_{i\varphi} R_k^u z_d^{(j-1)} u(t)\right|, \ \forall\varphi, i; \ \ \forall t \in [0,\infty). \tag{3.17}$$

Substitute inequalities (3.13) and (3.14) into inequality (3.16) and take into account the inequality (3.17). Thus the majorant for the functional sequence is $\left\{z_{d\rho}^{(j)}(t)\right\}$, $j = 2, 3, \ldots, \infty$ has the form

$$\left|z_{d\rho}^{(j)}(t)\right| \leqslant \left\{nl \max_{\psi,\varphi} |n_{\psi\varphi}| \max_k |R_k^u|\right\}^{j-1} \left|\sum_{\varphi=1}^{n}\sum_{k=1}^{l} n_{i\varphi} b_\varphi R_k^u\right| M, \ t \in [0, \infty).$$

4. BIBO CRITERION OF STABILITY OF A BILINEAR SYSTEM

Construct a majorant for the sequence z_d^j in the form of a geometric progression.

The first member of the progression

$$m_1 = \max_\varphi |b_\varphi| M.$$

Progression member with number "j"

$$m_j = \max_\varphi |b_\varphi|\, M q^{j-1},$$

where q is the denominator of the progression

$$q = nl \max_{\psi,\varphi} |n_{\psi\varphi}| \max_k |R_k^u|.$$

Sufficient condition for the convergence of a progression

$$nl \max_{\psi,\varphi} |n_{\psi\varphi}| \max_k |R_k^u| < 1. \tag{4.1}$$

Sufficient condition for the divergence of a progression

$$\exists n, l, \phi, \psi, k : nl \max_{\psi,\varphi} |n_{\psi\varphi}| \max_k |R_k^u| > 1. \tag{4.2}$$

The condition (4.1) guarantees the convergence of all numerical sequences of elements of the solution matrices of the generalized Lyapunov equation at each step in the iteration process. This means that if the condition of boundedness of the number M and the sufficient condition of convergence of the progression are satisfied, the bounded input provides a bounded output, which means BIBO stability of the bilinear system. A similar condition (4.2) means that there is at least one divergent progression on the finite interval under consideration, which allows constructing an iterative process for calculating an unbounded solution. Satisfaction of the condition (4.2) leads to BIBO instability of the bilinear system. According to the Weierstrass criterion, the sequences of partial sums m_j converge uniformly and absolutely. Note that the obtained conditions for BIBO stability of the bilinear system, in contrast to the sufficient criterion, allows one to analyze the dependence of the BIBO stability condition of the bilinear system not only on the amplitude of the input action, but also on its spectrum. In particular, these conditions include estimates of the modules of the residues of the image of the input function at the poles of the characteristic equation of the image.

Theorem 2. *For* (2.1) *with a linear part* (2.2) *defined on the real axis* $t \in [0, \infty)$, *Hurwitz matrix* A *with a simple spectrum, the bounded on the interval* $[0, \infty)$ *functions* $u_\gamma(t)$, *the inequalities*

$$\int\limits_0^\infty |u_\gamma(\tau)|\, d\tau \leqslant M_\gamma > 0, \ \ \gamma = 1, 2, \ldots, m,$$

an iterative procedure for constructing a solution of the system (2.1) *of the form* (2.6)–(2.7) *with zero initial conditions is also given, and a non-degenerate transformation of the coordinates of the system with matrix* T (3.2), *the spectral decompositions of the Volterra kernels of the solution of the original and transformed systems in the time and frequency domains of the form, at each iteration step, are valid*

$$z_{d\varphi}^{(j)}(s) = (s - s_\varphi)^{-1} \mathcal{L}\left[N z_{d\varphi}^{(j-1)}(\tau)\, u(\tau)\right],$$

$$\left|z_{d\rho}^{(j)}(t)\right| \leqslant \left\{nl \max_{\psi,\varphi} |n_{\psi\varphi}| \max_k |R_k^u|\right\}^{j-1} \left|\sum_{\varphi=1}^n \sum_{k=1}^l n_{i\varphi} b_\varphi R_k^u\right| M, \quad t \in [0, \infty).$$

When the condition (4.1) *is satisfied, the functional series converge to the solution absolutely and uniformly.*

5. CONCLUSION

The paper proposes new algorithms and a methodology for constructing a spectral iterative solution to a continuous bilinear equation, which are a development of the approach proposed in [24]. Compared to these works, the proposed approach has the following advantages:

1. The obtained criteria for the convergence of numerical sequences of elements of the solution of a bilinear equation in the time and frequency domains determine a guaranteed limited area of distribution of methods for solving and analyzing linear control systems to the class of bilinear systems for many applications.

2. New sufficient conditions for BIBO stability of bilinear systems are obtained and a new method for calculating the steady-state values of their solutions is proposed.

3. To construct and study the solution, instead of the multidimensional Laplace transform, it is proposed to use frequency methods based on the direct Laplace transform.

4. The obtained spectral decompositions of solutions by the spectrum of the dynamics matrix of the linear part, as well as the spectrum and residues of the images of impacts, allow us to estimate their influence on the stability and dynamic characteristics of the bilinear system.

5. For the special case of MIMO BTI equations of continuous systems with effects whose images are fractional rational functions converging on a finite interval, analytical formulas for the iterative construction of solutions are obtained.

The limitations of the proposed approach should be noted:

1. The linear part of the bilinear system must be stable, and its dynamics matrix must have a simple spectrum.

2. The study is limited to deterministic impacts.

3. The poles of the images of the effects must be in the left half-plane of the complex plane.

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